A reverse holdup problem[*]

When workers' lack of bargaining power slows economic adjustments

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Abstract

In a labor market with horizontal matching, we show that increasing workers' bargaining power can lead to higher wages, more jobs, better jobs and higher profits. Higher bargaining power increases incentives for workers to search, which in turn may increase some employers' incentives to switch to new production activities. However, incurred search costs could cut the surplus for a majority of workers, even when it creates jobs for the others and increases aggregate surplus. The median voter may thus object to the economic adjustments.

JEL: J3, J6, L1 **Keywords**: search, matching, bargaining power, unemployment

1   Introduction

Debates on labor market issues associated with the "Great Recession" often blame increasing and lasting unemployment on ageing production structures resulting from ineffective or misled industrial policies.[1] This ignores that causality may run both ways. We focus on the case in which causality runs from poor labor policies to an inefficient diversity of the production structure.

We assume workers to be of different (given) types, corresponding to horizontally differentiated skills. We argue that employers may not want to invest in a sector requiring workers with

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[1]For instance, during his campaing to become president of the European Commission, Jean-Claude Junker was calling for a "more effective European industrial policy (...) being felt across all EU policies that affect jobs and growth" (Speech by Jean-Claude Junker, EPP candidate for President of the European Commision, at the Nueva Economía Fórum, Madrid, 12 May 2014, retrieved from http://juncker.epp.eu)

minority skills if they know that the workers with these skills do not want to bear the search costs to find the right employer. This happens when the expected share of the surplus these workers get is not high enough. Hence, insufficient, rather than excessive, bargaining power for workers may explain the inability of an economy to create good jobs.

This issue is less visible when the economy is doing well, since the productivity of mismatched workers is still sufficiently high to avoid structural unemployment. However, even without unemployment, increasing workers' bargaining power can increase the quality of matching because it gives employers incentives to offer the right kind of jobs. Firms offset their loss of bargaining power by increasing their profits from a changed production mix resulting from improved matching in the labor market.

Increasing bargaining power also has distributional consequences: we find that a majority of workers may have an incentive to oppose an increase in workers' bargaining power. This is the case even if this increase can be linked to more jobs and better (matched) jobs. This rejection stems from the fact that the dominant group of workers may lose surplus in spite of their increased bargaining power. This surplus loss arises if the expected search cost they bear as a result of the diversification of the economy outweighs the gains from the wage increase.

The novelty of our results comes from the fact that investment decisions made by the firms determine the horizontal diversity of the production structure. Thus, increasing workers' bargaining power does not affect the quality, but the diversity of jobs offered. Hence, higher bargaining power may increase total employment by allowing jobless people to find a job in which they are productive enough, because it gives incentives to the firms to offer such jobs. Therefore, in contrasts to the number of authors who already made the point that increasing the minimum wage, unemployment benefits or workers' outside option may increase the quality of jobs by inducing workers to search (Albrecht and Axell, 1984; Marimon and Zilibotti, 1999; Acemoglu, 2001), we find that there is no necessary global tradeoff between more jobs and better jobs. However, because of the endogenous diversity of the production structure, a policy increasing the quality of matching and firms' profits may create winners and losers among the workers.

Our results are driven by the investment decisions of firms: too low a bargaining power for workers deters firms from making the right investment because workers don't get a sufficiently high share of the surplus to bear the cost to search for the right firm. This idea contrasts with the classic holdup problem where a high bargaining power for workers deters the employers to invest enough and therefore leads to suboptimal productivity (Acemoglu and Shimer, 1999). We, in turn, focus on the masses of workers who may not be qualified enough to easily find a job and

bear most of the search costs. This assumption obviously does not apply to the market for the most highly qualified jobs, but is quite relevant for the large number of young workers having a hard time finding a job - whether in Southern Europe, in the Middle East or many emerging economies.[2] The intuition behind the benefits from higher bargaining power for workers is reminiscent of Farrell and Gibbons (1995). They show that buyers are willing to better communicate their preferences to sellers if their bargaining power is sufficiently high. Hence, both buyers and sellers may benefit from an increase in buyers' bargaining power, because it allows sellers to make investments that increase the total surplus from the trade.[3]

We also assume workers have horizontally differentiated skills. This is quite a widespread situation. In a recent paper, Papageorgiou (2014) shows that for a representative sample of the US population, the data "favours the model of comparative advantage as opposed to the one-dimensional model of ability, which is most often used in the literature" (p.2). Previous research also insisted on the importance of horizontal matching between the skills of workers and the needs of firms (Brueckner et al., 2002; Hamilton et al., 2000; Helsley and Strange, 1990). An important constraint for employers is that, by choosing a sector, they have to make an ex ante (costless) investment that restrains their hiring possibilities.

We are interested in unemployment originating from the failure of the market to supply the right diversity of jobs. To clearly identify this relationship, we rule out the possibility of a general scarcity of jobs and therefore assume firms are able to hire several workers with constant returns to scale. In that sense, our model is very close to Albrecht and Axell (1984).[4] By contrast, the assumption of a limited capacity of firms to hire is key to understanding many of the important results in search theory (Hosios, 1990; Mortensen and Pissarides, 1994, 1999), where there exists a tradeoff between the resources used to advertise jobs (too many vacancies) and the costs of leaving workers unemployed (not enough vacancies).

For the main part of the paper, we treat bargaining power as an exogenous parameter, identical across workers and independent of the jobs actually offered. The main idea behind this specification is that there exist country-level regulations that determine the respective "formal" power of employers and workers in a wage negotiation. This is for instance determined by the legislation on

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[2]Changing this assumption would turn the problem upside down, as employers need to have sufficiently high bargaining power to look for the right workers, and workers pursue diversified education only if they expect employers will bear the search costs to find them. In that case, our result would thus be that the most qualified workers need not to have too high a bargaining power for the market to provide sufficiently diverse jobs and skills.

[3]Our results also relate to another ex ante investment problem documented by Gall et al. (2006), where limits in the allocation of surplus within firms may either hinder the investment in human capital from workers, or generate a misallocation of employers to employees.

[4]The main difference with this model is that we assume all firms have the same ex ante productivity, and only differ in their chosen sector.

contracts and, in many countries, sector and nation-wide negotiations between unions, employers'
representatives and the government. It is in that sense that we consider bargaining power to be the
result of an agreement between the median voter and firms.

Motivating example

We believe this model can help in understanding the failure of labor policies in the large number
of European countries which have exceptionally high unemployment rates among their educated
youth.

Assume there is an 'old' sector, in which a majority of older workers are more productive
(because, for instance, they are more experienced), and a 'new' sector, where a minority of young
workers are more productive (because their skills are more adapted). In the two political institu-
tions that may influence workers' bargaining power (the unions and the government), the median
voter is an older worker. Hence, a policy that would make both employers and younger workers
better off by creating more and better jobs, might never be implemented, unless those 'young'
workers manage to get more political influence. Spain provides a particularly striking example of
our main idea. Before the Great Recession, the Spanish job market was offering low bargaining
power to workers without an existing long-term contract. In the period 1995-2001, more than 90%
of new hires were under temporary contracts, such that:

> *(...) In just a decade, a fairly regulated labour market with high dismissal costs
> and strong unions' bargaining power at wage determination turned into a very di-
> visive labour market, where around two-thirds of the employees enjoyed permanent
> contracts (...) and kept the high bargaining power of the past, while the remaining
> one-third are workers under fixed-term contracts entailing much less favourable em-
> ployment conditions.* Dolado et al. (2002)

At the time, economic conditions were favourable and Spain enjoyed a high level of employ-
ment, including among young workers. However, those workers[5] had low bargaining power and
were largely working in 'old' sectors (construction work was one of the main drivers of job cre-
ation). With the crisis, the productivity of young workers became insufficient to be employed in
those sectors, and they were the most heavily hit by unemployment.[6] As a consequence of the

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[5]Well before the crisis, young educated workers were called 'mileuristas' because no matter their skills and sector
of employment they had a standardised starting salary of 1000 € and only very slowly progressed out of that category.

[6]Spain's youth unemployment (less then 25 year old) rose from 18.2% in 2007 to 53.2% (+35.0%) in 2012. For the
rest of the population, the figure rose from 7.0% to 19.4% (+12.4%). This can be compared with the average of the 27

crisis, the government's policy has been to implement labor market reforms such that "the main measures of the 2012 reform basically amount to a substantial shift in bargaining power away from workers and towards employers" (Bentolila et al., 2012). In our setting, this policy (together with a decrease in the outside option of workers) corresponds to an attempt to increase employment by making workers employable in the wrong sector. We argue that such a policy is likely to worsen the main problem of the country: the economic structure. We also argue that this policy is the most likely to be implemented in practice if the median voter is not compensated from the more dynamic labor market that would arise with higher workers' bargaining power.

Outline

We present the setup of the game in the next section, and its resolution in Section 3. The baseline model is static, and therefore employers do not bear any opportunity cost. In Section 4, we provide the main results of a dynamic model. When workers search, the opportunity cost of waiting for the right workers to come is higher for firms in the sector corresponding to minority skills. The equilibrium share of the jobs corresponding to the skills of the minority is therefore lower than the share of the population with those skills. Hence, the bargaining power needed for those workers to search in equilibrium is higher. The dynamic setting allows us also to formalize the tradeoff between frictional unemployment generated by search and structural unemployment generated by the inability of the economy to provide diversified jobs. For the rest, the results are similar to those in the static specification.

In Section 5, we assume that a worker can negotiate a higher wage if he can easily find a firm corresponding to his type than if he has to bear a high search cost to find a good match.[7] This strategic bargaining on wages provides another reason for the median voter to oppose an increase in his formal bargaining power. If, by doing so, the diversity of jobs offered increases, workers of the majority type bear two additional costs. The first one is the increase in the search costs discussed above. The second is the fact that the lower share of firms in their sector decreases their outside option, and therefore the wage they can bargain. This also implies that workers of the minority type always receive a lower wage than workers of the majority type, for a given matching quality.

Section 6 presents additional results that emphasize the differences between the minimum

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countries of the European Union, where youth unemployment increased from 15.7% to 22.9% (+7.2%) and from 6.6% to 10.1% (+3.5%) for the rest of the population (Eurostat, 2013).

[7]This corresponds to the idea that competition among firms is an important factor of wage determination (Cahuc et al., 2006).

wage, outside option and bargaining power. We also study a policy of on-the-job training, present the basic results with more than two sectors, and allow for asymmetric bargaining power among workers. We conclude in Section 7. Formal proofs are collected in the Appendix.

2 Setup

The economy is composed of two groups, each of them being a continuum of mass 1. The first group is the workers, with an exogenous fraction α of type a and $1 - \alpha$ of type b. We assume a strict majority of workers are of type a, $\alpha \in (\frac{1}{2}, 1)$.[8] The share of workers of each type is common knowledge and the types are perfectly observable. The second group is composed of ex-ante identical employers, who endogenously decide to offer jobs in sector A or B at no cost. A good match ($a - A$ or $b - B$) generates surplus V, and a mismatch generates surplus v, with $V > v > 0$. An employer can be matched with more than one worker, but a worker can work for either zero or one employer.[9] When a worker is successfully matched with an employer, he keeps a share $\lambda \in (0, 1)$ of the surplus, corresponding to his bargaining power. A worker also benefits from an outside option of value r.

In the baseline version of the model, the game is sequential and played only once. We solve by backward induction and look for Subgame Perfect Nash Equilibria (SPNE). As we want to identify market failures that are not coordination failure, we allow employers to coordinate towards their Pareto-dominant equilibria. We do this to avoid the case where a diversified production structure is an equilibrium, but coexists with a somehow trivial equilibrium where jobs are only provided in one sector and all employers make lower profit. We also want to rule out the case where all employers offer jobs in the sector corresponding to the skills of the minority of workers. In the first stage, employers simultaneously choose their sector. We denote by γ the resulting fraction of the employers in sector A, and $1 - \gamma$ the fraction of the employers in sector B. As employers are free to choose their sector, the expected profit is identical for all firms. In the second stage, each worker gets a first free match.[10] Then, he decides either to search (S) for another employer (the linear cost for each search is $s > 0$), to accept the job (Y) (and take a share λ of the surplus), or

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[8]We briefly explain in Section 6.3 why the logic is similar with a larger number of types.

[9]We therefore assume constant returns to scale. We also implicitly assume a small open economy not influencing the world prices by its production structure.

[10]This is not a crucial assumption. The conditions for workers to search for the right job remain identical, and the condition for a worker to enter a market when no employer is in his sector would simply become $\lambda v - s \geq r$ instead of $\lambda v \geq r$. As the share of firms in each sector γ is common knowledge before workers make their decisions, our results are qualitatively unaffected. The payoff of each worker would be the payoff we compute minus one initial search cost s if the worker enters the market, and the payoff of a worker staying out of the market would still be equal to r.

to leave the market (L) and take his outside option r. Workers know the fraction of employers in each sector, but cannot direct their search. Employers and workers are risk neutral, and therefore maximize their expected utility.

In this static version of the model, the only source of unemployment corresponds to workers refusing to work when offered a job. Search costs are paid by workers but do not delay the time of employment (this "frictional" unemployment is introduced in Section 4). To make the results nontrivial, we assume $\lambda V > r$. The first case we present corresponds to an economy without unemployment ($\lambda v \geq r$), as workers always prefer a bad match over being unemployed. Then, we study a second case ($\lambda v < r$), where unemployment is possible, as workers prefer to remain unemployed than to accept a bad match.

3 Equilibrium in the static economy

3.1 An economy without unemployment

a. Second stage

We start by assuming $\lambda v \geq r$, so that workers always prefer a bad job over unemployment. In the second stage, the relevant decision of a worker is thus between accepting the first job or searching for a better one. The expected payoff of a worker of type i offered a job in the right sector is

$$U_{i,I} = \lambda V, \tag{1}$$

and as there is no better match for him, this worker never wants to search. The expected payoff of a worker of type a offered a job in sector B is given by

$$U_{a,B}(Y) = \lambda v$$
$$U_{a,B}(S) = \lambda V - \frac{s}{\gamma} \tag{2}$$

Depending on whether he accepts the job (Y) or searches (S), where $\frac{s}{\gamma}$ is the expected search cost to be paid before meeting an employer of sector A. Hence, a worker of type a matched with an employer of sector B decides to search if his expected utility by searching is higher than his

utility from accepting the job, this is if:

$$\gamma \geq \frac{s}{\lambda(V-v)} \tag{3}$$

Similarly, a worker of type *b*, matched with an employer of sector *A* decides to search if his expected utility by searching is higher than his utility of accepting the job, that is, if:

$$1 - \gamma \geq \frac{s}{\lambda(V-v)} \tag{4}$$

b.   First stage

We focus on employers' best response to the four possible pure strategies of workers in the second stage: (i) no one searches, (ii) both types of workers search, (iii) only workers of type *a* search, and (iv) only workers of type *b* search. We show here that two equilibria are on the Pareto frontier of employers: either all jobs offers are in sector *A* and no one searches, or jobs are offered in both sectors and all workers search. Offering all jobs in the sector corresponding to the minority of workers is never beneficial for employers.

(i) No one searches

In this case, the respective expected profits are given by:

$$E(\pi|A) = \alpha(1-\lambda)V + (1-\alpha)(1-\lambda)v \tag{5}$$

$$E(\pi|B) = (1-\alpha)(1-\lambda)V + \alpha(1-\lambda)v. \tag{6}$$

As $\alpha > \frac{1}{2}$, it is always a best response for all employers to choose sector *A*, so that $\gamma = 1$. Obviously, if there are only employers in sector *A*, no one searches. This is a SPNE if there is no profitable deviation from employers. This is always true as there is a continuum of employers. Hence, a single (atomless) firm switching to sector *B* is never enough to make workers of type *b* search.

(ii) Both types of workers search

In this case, the expected profit of an employer in sector *A* is given by the sum of the expected profit from workers of type *a* for which he is a first draw, and the one from workers of type *a* who

found him after searching. This is:

$$E(\pi|A) = \frac{\alpha}{\gamma}(1-\lambda)V. \tag{7}$$

Similarly, for an employer in sector B:

$$E(\pi|B) = \frac{(1-\alpha)}{1-\gamma}(1-\lambda)V. \tag{8}$$

In equilibrium, the expected profits must be identical. Therefore, at such an equilibrium, $E(\pi|A) = E(\pi|B)$, $\gamma = \alpha$. For this to be a SPNE, we need to actually have both types of workers searching when $\gamma = \alpha$ in the second stage. As $\alpha > \frac{1}{2}$, if workers of type b search, both types of workers search. The condition therefore directly derives from equation (4):

$$1 - \alpha \geq \frac{s}{\lambda(V-v)} \tag{9}$$

$$\lambda \geq \frac{s}{(1-\alpha)(V-v)} = \lambda' \tag{10}$$

This means that workers' bargaining power must be high enough, such that the surplus from finding a good job is higher than the expected search cost.

(iii) If only workers of type a search

Lemma 1 *There is no SPNE where only workers of type a search and $\gamma < 1$.*

Proof. As $\alpha > \frac{1}{2}$, even without search from workers of type a, the expected profit of an employer is higher in sector A when workers of type b do not search. If only workers of type a search, the respective profits of both types of firms are given by

$$E(\pi|A) = \frac{\alpha}{\gamma}(1-\lambda)V + (1-\alpha)(1-\lambda)v \tag{11}$$

$$E(\pi|B) = (1-\alpha)(1-\lambda)V. \tag{12}$$

As $\alpha > \frac{1}{2}$ and $\gamma \leq 1$, $\frac{\alpha}{\gamma}(1-\lambda)V + (1-\alpha)(1-\lambda)v > (1-\alpha)(1-\lambda)V$ and no firm wants to choose sector B. Thus, there is no need for workers of type a to search as $\gamma = 1$ (search is an out-of-equilbrium strategy). ∎

(iv) If only workers of type b search

In this last case, the respective expected profits are given by:

$$E(\pi|A) = \alpha(1-\lambda)V \tag{13}$$

$$E(\pi|B) = \alpha(1-\lambda)v + (1-\alpha)\frac{(1-\lambda)V}{1-\gamma}. \tag{14}$$

The condition of isoprofit is thus met when:

$$(1-\gamma) = \frac{(1-\alpha)V}{\alpha(V-v)}. \tag{15}$$

A first obvious example is a corner solution where all firms are active in Sector B ($\gamma = 0$). Indeed, as long as $\alpha < \frac{V}{2V-v}$, $E(\pi|A) < E(\pi|B)$, $\forall \gamma \in (0,1)$. This case is a mirror of the case (i), and search for workers of type b is an out-of-equilibrium strategy. It is however Pareto dominated for employers, as their expected payoff $\alpha(1-\lambda)v + (1-\alpha)(1-\lambda)V$ is lower than if all were active in sector A.

For the considered consumer behaviour to be part of a SPNE it must be true that, for the value of γ defined in equation (15), only workers of type b search. This is the case when the two following conditions are simultaneously met:

$$\lambda \geq \frac{\alpha s}{(1-\alpha)V} \tag{16}$$

$$\lambda < \frac{\alpha s}{\alpha(V-v) - (1-\alpha)V}, \tag{17}$$

where equations (16) and (17) are obtained by replacing γ in equations (3) and (4) by the value found in (15). This equilibrium may coexist with one where both types of workers search (not all the time, as condition (16) is less restrictive than condition (10)). The expected profit is equal to $(1-\lambda)V > \alpha(1-\lambda)V$, and this last equilibrium is thus never on the Pareto frontier of firms.

(v) Summary

Equilibrium	Expected Profit	Condition 1	Condition 2
No one searches, $\gamma = 0$	$(1-\lambda)((1-\alpha)V + \alpha v)$		
No one searches, $\gamma = 1$	$(1-\lambda)(\alpha V + (1-\alpha)v)$		
Both types of workers search, $\gamma \in (0,1)$	$(1-\lambda)V$	$\lambda \geq \frac{s}{(1-\alpha)(V-v)}$	
Workers of type b only search, $\gamma \in (0,1)$	$(1-\lambda)\alpha V$	$\lambda \geq \frac{\alpha s}{(1-\alpha)V}$	$\lambda < \frac{\alpha s}{\alpha(V-v)-(1-\alpha)V}$

Table 1: Baseline model without unemployment

The following lemma follows from the above results.

Lemma 2 *In equilibrium, if firms are allowed to coordinate towards a Pareto superior equilibria, either all firms are active in Sector A and no workers search, or a share $\gamma = \alpha$ of firms are active in sector A and both types of workers search. The latter happens if workers' bargaining power is sufficiently high, $\lambda \geq \frac{s}{(1-\alpha)(V-v)} = \lambda'$.*

The proof directly follows from the expected profit and existence conditions in the different equilibria. Those results are summarized in Table 1.

c. Results

Our analysis therefore focuses on a comparison between an equilibrium with firms in sector *A* only and an equilibrium with firms in both sectors and workers searching for a good match. A first general assessment of the respective efficiency of these equilibria is to measure the aggregate surplus generated by the labor market, without considering the distribution. The main advantage of an equilibrium with only firms in sector *A* is that there is no search cost paid in equilibrium. The main advantage of a diversified equilibrium is that all workers end up with a good match. Hence, it is easy to show that, following this criterion, an equilibrium with diversity should be preferred when:

$$\alpha V + (1-\alpha)v < V - \alpha \frac{1-\gamma}{\gamma}s + (1-\alpha)\frac{\gamma}{1-\gamma}s \tag{18}$$

$$s < (1-\alpha)(V-v). \tag{19}$$

The right hand side is decreasing in α for values of $\alpha \in (\frac{1}{2}, 1)$, the intuition being that the benefits from diversification are smaller when the share of workers of the minority type is smaller. In particular, diversification never increases aggregate surplus when $s > \frac{V-v}{2}$. There exist a widespread intuition in economics that higher bargaining power for workers may create better jobs, at the cost of higher unemployment and lower profit for employers. Hence, a social planner maximizing aggregate welfare should increase workers' bargaining power to any level that allows sustaining search as an equilibrium when s is sufficiently small, even when it means transferring surplus from employers to workers. However, we can show that increasing workers' bargaining power is not a simple transfer among players that increases total welfare. Employers can gain from workers' bargaining power, and workers themselves can lose.

This intuition is illustrated in Figure 1, using parameter values of $v = 2$, $V = 3$, $s = 0.3$, $\alpha = 0.55$ and $r = 0$. The top-left figure illustrates how the average wages evolve with λ. The first trend is linear, when bargaining power increases, all wages increase. The second is a discontinuity
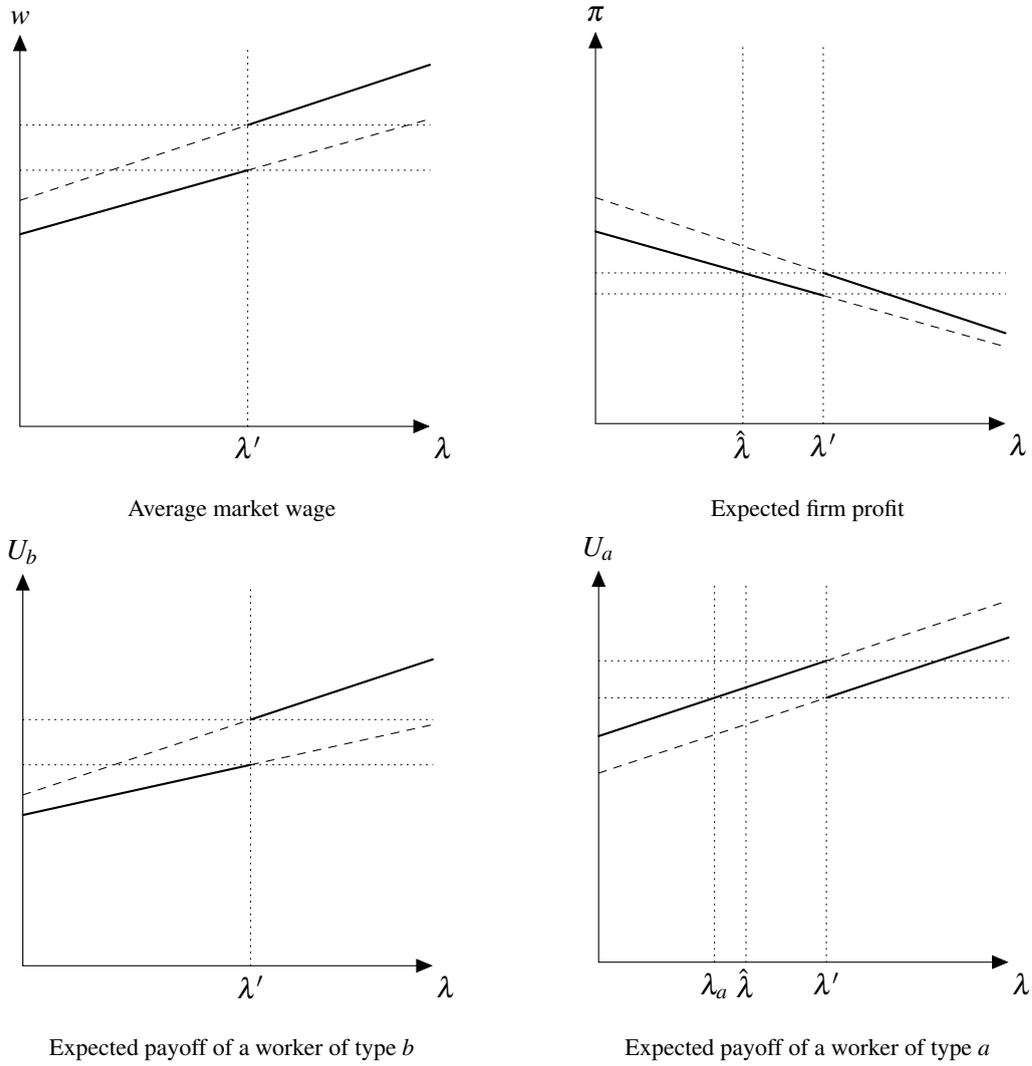
Average market wage

Expected firm profit

Expected payoff of a worker of type *b*

Expected payoff of a worker of type *a*

Figure 1: Threshold values of λ when $\lambda v > r$.

at λ'. Whenever λ reaches this level, workers start searching for a good match, and wages are therefore higher. The top-right corner shows that the increase in bargaining power can benefit firms. While the linear trend for firms is that higher bargaining power for workers decrease their expected profit, this profit discretely increases at λ', as employers benefit from a higher matching quality. Thus, one can identify $\hat{\lambda}$, such that an increase in workers' bargaining power allowing for a better matching quality benefits employers if the magnitude of this increase is lower than $\lambda' - \hat{\lambda}$. The bottom-left corner represents the expected payoff of a worker of minority type b. The expected payoff of this worker discretely increases when his bargaining power reaches λ'. Indeed, at this point this worker is made indifferent between searching and accepting a first mismatch. But as γ becomes lower than 1, there are also cases where this workers does not even need to search, and is therefore made strictly better off. Finally, the bottom-right corner represents the expected payoff of a worker of type a. For this worker, crossing the value λ' represents a discrete loss. Indeed, at this point, workers of the majority type start to have to bear search costs, and are thus made worse off despite their gain in bargaining power. Hence, one can determine a threshold value λ_a such that for a worker of the majority type to accept an increase in bargaining power that changes the economic structure, the magnitude of this increase should be at least higher than $\lambda' - \lambda_a$. In this precise example (but this is not always the case), as $\lambda_a < \hat{\lambda}$, there is no possible deal that satisfies both employers and the median worker, even though an increase of bargaining power from any $\lambda < \lambda'$ to any $\lambda \geq \lambda'$ increases aggregate welfare.

We formalize this result in the following Proposition, considering an increase of the bargaining power from $\lambda < \lambda'$ to exactly λ'.

Proposition 1 *If workers never choose to remain unemployed, setting workers' bargaining power to λ' increases both the average wage and the (expected) profit of employers when (i) the initial value of λ is low enough, such that jobs are only offered in sector A, $\lambda < \lambda'$ and (ii) the initial value of λ is high enough ($\lambda > \hat{\lambda} = 1 - \frac{((1-\alpha)(V-v)-s)V}{(\alpha V + (1-\alpha)v)(1-\alpha)(V-v)}$), such that increasing the quality of matching compensates for employers' decreased bargaining power. However, (iii) for any pair $\{\hat{\lambda}, \lambda'\}$ such that increasing workers' bargaining power from any $\lambda \in (\hat{\lambda}, \lambda')$ to λ' increases employers' profits, there exists a value $\lambda_a = \frac{s(\alpha V - (1-\alpha)^2(V-v))}{\alpha V(1-\alpha)(V-v)} < \lambda'$ such that, for any $\lambda \in (\lambda_a, \lambda')$, workers of type a are better off with bargaining power λ than λ'.*

We show how these threshold values evolve with α in Figure 2, for parameters value such that, most of the time, $\hat{\lambda} < \lambda_a$. In the zone between λ' and $\hat{\lambda}$, an increase in the bargaining
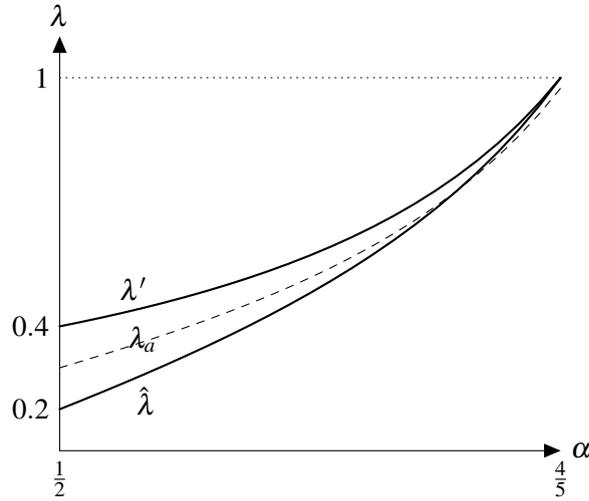
Figure 2: Threshold values of λ for $v = 1, V = 2, s = 0.2, r = 0$.

power to λ' always increases the profit of employers. When the initial level of bargaining power is close to $\hat{\lambda}$, the only way to make employers better off is to set exactly λ'. When the initial level of bargaining power is higher, even higher values of λ increase firms' expected profit. Hence, this result emphasizes a somehow classic tradeoff in search models between the quality of jobs and the intensity of search. However, the novelty is that the game creates winners and losers among workers, as the very structure of the economy changes, with new industries appearing and some workers bearing additional search costs. The group that may hinder a reform of the labor market towards higher bargaining power for workers is precisely the largest group of workers. For instance, if workers are organized in unions, the workers of the majority type may have no incentive to support a policy that increases their expected search costs, even if it increases aggregate welfare and increases their own wage.

This problem is not easy to solve for minority type workers. Assume these workers decide to lobby, alone, for an increase in their bargaining power. On the one hand, this may increase their willingness to search for a given number of employers in sector B. On the other hand, this will decrease employers' expected profit in sector B, as the surplus extracted by employers from a good match in sector A becomes higher than the surplus they could extract in sector B. Therefore, the share of employers in sector B would be even lower than $1 - \alpha$. This implies a higher search cost to be paid by a minority of workers, and the need for an even higher bargaining power for them. Hence, if one needs to compensate the median voter for the increase in his search cost, this can only be done by (lump sum) money transfers that affect neither the type b workers' incentives to search, nor employers' incentive to offer jobs in sector B. In the next subsection, we show how

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this all evolves when there is room for structural unemployment, this is if workers prefer to remain unemployed than to accept a "bad" job.

3.2 An economy with potential unemployment

We now study the case where $\lambda v < r$, so that workers always prefer unemployment over a bad job. In the second stage, the relevant decision of a mismatched worker is thus between staying unemployed or searching for a good match. The expected payoff of a worker of type i offered a job in the right sector is

$$U_{i,I} = \lambda V, \tag{20}$$

and as there is no better match for him, this worker never wants to search. The expected payoff of a worker of type a offered a job in sector B is given by

$$\begin{aligned} U_{a,B}(L) &= r \\ U_{a,B}(S) &= \lambda V - \frac{s}{\gamma}, \end{aligned} \tag{21}$$

where r is the outside option received when choosing to refuse a mismatch and leave (L) the market. Following the same logic as in the case without unemployment, we can solve the conditions of existence for the four possible Nash equilibria, summarized in Table 2 (see Appendix 2 for detailed computations). In particular, we identify the threshold value of $\lambda = \lambda''$ such that both type of workers start searching in equilibrium. It corresponds to the value of λ at which a worker of type b will begin to search. When $\gamma = \alpha$, this is:

$$\lambda'' = \frac{(1 - \alpha)r + s}{(1 - \alpha)V}. \tag{22}$$

A first difference with the model without unemployment is that we cannot rule out the equilibrium where workers of type b only search, because employers are indifferent between this equilibrium and the one where no one searches and $\gamma = 1$. This equilibrium, however, does not affect the following proposition, as it implies a profit for firms identical to the one in the equilibrium with $\gamma = 1$, and lower surplus for workers (with identical unemployment level, but strictly positive equilibrium search costs). A second difference is that now, increasing bargaining power also decreases unemployment. Indeed, with $\lambda < \lambda''$, there is a strictly positive level of unemployment (equal to $1 - \alpha$). When bargaining power increases above λ'', there is no unemployment. The tradeoff is

between the payment of search costs and the level of unemployment. In a dynamic setting (as in the next section), search is also a source of unemployment, and the tradeoff is between unemployment generated by search (frictional) and the one generated by the failure of the production sector to provide jobs corresponding to the skills of the minority (structural).

Equilibrium	Expected Profit	Condition 1	Condition 2
No one searches, $\gamma = 0$	$(1-\lambda)(1-\alpha)V$		
No one searches, $\gamma = 1$	$(1-\lambda)\alpha V$		
Both types of workers search, $\gamma \in (0,1)$	$(1-\lambda)V$	$\lambda \geq \frac{(1-\alpha)r+s}{(1-\alpha)V}$	
Workers of type b only search, $\gamma \in (0,1)$	$(1-\lambda)\alpha V$	$\lambda \geq \frac{\alpha s+(1-\alpha)r}{(1-\alpha)V}$	$\lambda < \frac{\alpha s+(2\alpha-1)r}{(2\alpha-1)V}$

Table 2: Baseline model with unemployment

Proposition 2 *If workers prefer unemployment over a mismatch, setting workers' bargaining power to λ'' increases both the average wage and the (expected) profit of employers, and decreases unemployment when (i) the initial value of λ is low enough, such that jobs are only offered in sector A, $\lambda < \lambda''$ and (ii) the initial value of λ is high enough ($\lambda > \hat{\lambda}'' = 1 - \frac{(1-\alpha)(V-r)-s}{(\alpha(1-\alpha)V)}$), such that increasing the quality of matching compensates for employers' decreased bargaining power. However, (iii) for any pair $\{\hat{\lambda}'', \lambda''\}$ such that increasing workers' bargaining power from any $\lambda \in (\hat{\lambda}'', \lambda'')$ to λ'' increases employers' profits, there exists a value $\lambda_a'' = \frac{(1-\alpha)\alpha r+((3-\alpha)\alpha-1)s}{(1-\alpha)\alpha V} < \lambda''$ such that, for any $\lambda \in (\lambda_a'', \lambda'')$, workers of type a are better off with bargaining power λ than λ''.*

This second Proposition is illustrated in Figure 3. The only zone relevant to this subsection is located between the two horizontal lines, as they correspond to the two conditions $\lambda V \geq r$ and $\lambda v < r$. Compared to the case without unemployment presented in Figure 2, we observe that the condition for an increased λ to make employers better off is much weaker (because employers receives zero surplus of a mismatch), while the necessary level of bargaining power to incentivize workers to search is higher (because it depends on their outside option, higher than the payoff they get from a mismatch). As in Proposition 1, given there are several types of workers with different payoffs, the composition of the workforce matters and workers of the majority type *a* may oppose an increase in their own bargaining power.
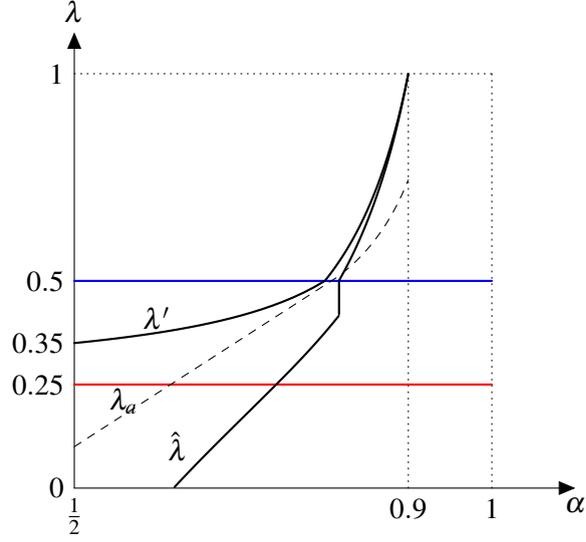
Figure 3: Threshold values of λ for $v = 1$, $V = 2$, $s = 0.1$, $r = 0.5$.

4   A dynamic setting

In this section, we show that the two Propositions of the baseline model are robust to a dynamic version of the model.[11] We also formalize a tradeoff between the frictional unemployment generated by search and the structural unemployment coming from the failure of employers to offer diversified jobs. While we still assume that, at each period, there is a continuum of mass 1 of workers and employers, the stock of workers is regularly renewed. Time is accounted in fixed periods and firms exist in perpetuity. At the end of each period, existing workers exit the economy with probability $\tau \in (0,1)$, and a continuum of new workers of mass τ enters the market. A match between a firm and a worker still provides a payoff v or V to be shared with a bargaining power λ for the worker. However, this payoff is received at every period when the two players are matched. We focus this presentation on the case where there is potential for structural unemployment $\lambda v < r$, so that workers remain unemployed while searching for a job. We derive the (very similar) results with $\lambda v \geq r$, and therefore on-the-job search, in Appendix 4.

A worker that decides to search while being mismatched still receives his outside option, but bears a per-period search cost s. As we have assumed $\lambda v < r$, he always prefers unemployment over a mismatch. For the same reason as in the baseline model, the two considered equilibria either imply $\gamma = 1$ and jobs in sector A only, or jobs in both sectors and the two types of workers searching.

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[11]The notation of our dynamic setting is largely borrowed to Albrecht and Axell (1984).

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We first study the steady-state decisions of the workers, then those of the employers. Finally, we show that the preferences of the workers of type *a*, and thus the median voter, display the same properties as in Propositions 1 and 2: he may oppose an increase in his own bargaining power, while employers support it.

a. Workers

a..1 If no one searches and $\gamma = 1$,

Workers of type *a* accept an offer in the first period and keep their jobs, while workers of type *b* remain unemployed. Hence, a worker of type *a* receives an expected lifetime surplus of

$$E(U_a) = \frac{\lambda V}{\tau}, \tag{23}$$

and a worker of type *b*

$$E(U_b) = \frac{r}{\tau}. \tag{24}$$

a..2 If all workers search and $\gamma \in (0,1)$

Using the same reasoning as in the static setting, we know that if $\gamma > \frac{1}{2}$, and workers of type *b* search, workers of type *a* always search.

A worker of type *b* searches after being matched with a firm in sector *A* if the expected surplus from searching is higher than the surplus of keeping the "wrong" job. Denote by W^* the continuation value of searching,

$$W^* = r + (1 - \tau)(1 - \gamma)\frac{\lambda V}{\tau} - s + \gamma(1 - \tau)W^* \tag{25}$$

While searching, a worker keeps his outside option *r*. This worker survives in the next period with a probability $1 - \tau$ and receives a good match after searching once with a probability $1 - \gamma$. The continuation value of a good match is $\frac{\lambda V}{\tau}$, and the search cost is *s*. Finally, the worker survives and is matched again with a firm of the wrong type with probability $\gamma(1 - \tau)$.

For a worker to search, the continuation value W^* must be higher than the expected lifetime value of the outside option $\frac{r}{\tau}$. Hence, using equation (25) the condition for workers of type *b* to

search is

$$\frac{r}{\tau} \leq W^*$$

$$\Leftrightarrow \frac{r}{\tau} \leq \frac{(r-s)\tau + (1-\tau)(1-\gamma)\lambda V}{\tau(1-\gamma(1-\tau))}, \tag{26}$$

which gives us the condition λ' for workers of type b to search,

$$\lambda \geq \frac{r(1-\gamma)(1-\tau) + s\tau}{(1-\gamma)(1-\tau)V} = \lambda'. \tag{27}$$

b. Employers

Moving to the employers' side, we first determine the equilibrium value of γ when both types of workers search. Then we compare the profit of the employers under $\gamma = 1$ and no search, and the profit when both types of workers search. By doing so, we determine the minimum initial value of bargaining power $\hat{\lambda}$ such that firms are better off with $\lambda = \lambda'$ than with $\lambda \in (\hat{\lambda}, \lambda')$.

b..1 When both types of workers search   The expected profit of a firm in sector A comes from the workers of type a who have found the firm, either immediately or after searching (and keep their jobs until they disappear from the market). Those workers provide a per-period surplus $(1-\lambda)V$ to the employer. Hence, the expected surplus received by a firm operating in sector A is

$$E(\pi|A) = \tau[(\frac{\alpha}{\tau} + \frac{\alpha(1-\gamma)(1-\tau)}{\tau} + \frac{\alpha(1-\gamma)^2(1-\tau)^2}{\tau} + ...)](1-\lambda)V \tag{28}$$

This rewrites

$$E(\pi|A) = \frac{\alpha(1-\lambda)V}{\tau(1-(1-\gamma)(1-\tau))} \tag{29}$$

Following the same logic, we find the expected profit of firms in sector B,

$$E(\pi|B) = \frac{(1-\alpha)(1-\lambda)V}{\tau(1-\gamma(1-\tau))}. \tag{30}$$

The equilibrium share of firms in sector A, γ^* is given by equating the two expressions, $E(\pi|A) = E(\pi|B)$. This yields

$$\gamma^* = \frac{\alpha - \tau(1-\alpha)}{1-\tau}. \tag{31}$$

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It is interesting to note that, in this dynamic setting, $\gamma^* \geq \alpha$. This is because firms bear a part of the search cost: the opportunity cost of waiting for the right workers to find them. For this reason, for an interior solution $\gamma \in (0,1)$ to exist, α must not be too high, $\alpha \leq \frac{1}{1+\tau}$.

b..2 When no worker searches As there is a constant stock of workers of mass 1 and workers do not search, the expected profit of a firm is

$$E(\pi|A, \gamma = 1) = \frac{\alpha(1-\lambda)V}{\tau}. \tag{32}$$

b..3 Firms benefit from an increase in workers' bargaining power We want to determine the value of $\lambda = \hat{\lambda}$ such that firms are better off with $\lambda = \lambda'$ than with $\lambda \in (\hat{\lambda}, \lambda')$. To do so, it is useful to first determine the equilibrium profit $E(\pi|A) = E(\pi|B)$, when $\gamma = \gamma^*$ and $\lambda = \lambda'$. By replacing the value of γ found in (31) in (29), we obtain

$$E(\pi|\gamma^*) = \frac{(1-\lambda)V}{\tau(1+\tau)}. \tag{33}$$

and $\hat{\lambda}$ is found as the solution to $E(\pi|A, \gamma = 1) = E(\pi|\gamma^*, \lambda')$. This simplifies to

$$\hat{\lambda} = \frac{(1-\gamma)(1-\tau)r + s\tau - (1-\gamma)(1-\tau)(1-\alpha(1+\tau))V}{\alpha(1-\gamma)(1-\tau^2)V} \tag{34}$$

Therefore, we can write a proposition almost identical to the first two parts of Proposition 2 in the static setting.

Proposition 3 *If workers prefer unemployment over a mismatch, and if there exists a value $\lambda' \in (0,1)$ such that, for all values of $\lambda \geq \lambda'$ with $\gamma^* \in (0,1)$ workers search for their preferred job in equilibrium, then there exists $\hat{\lambda} < \lambda'$ such that increasing workers' bargaining power from any $\lambda \in (\hat{\lambda}, \lambda')$ to λ' (i) increases the quality of jobs and (ii) increases the expected profit of firms.*

Proof. If there is an interior solution $\lambda' \in (0,1)$, we can show that $\hat{\lambda} < \lambda'$. Indeed, using equations (27) and (34) $\hat{\lambda} < \lambda'$, as long as

$$\tau \leq \frac{(1-\gamma)(V-r)}{s + (1-\gamma)(V-r)}, \tag{35}$$

and we can show using the value of γ^* found in equation (31) and plugging it into equation (27) that the condition for an interior solution $\lambda \in (0,1)$ to exist is also $\tau \leq \frac{(1-\gamma)(V-r)}{s+(1-\gamma)(V-r)}$. ∎

A novelty with respect to the baseline model however is that the impact of search on unemployment is non trivial. Indeed, when $\lambda < \lambda'$, the expected level of unemployment is equal to $1 - \alpha$, the share of workers of type b, who choose to remain unemployed instead of being mismatched. When $\lambda > \lambda'$, and if there exists an interior solution for λ', the aggregate level of unemployment is given by the share of the population searching for a job at a given period. This can be decomposed as follows

$$u = \alpha \frac{(1 - \gamma)\tau}{1 - (1 - \gamma)(1 - \tau)} + (1 - \alpha) \frac{\gamma \tau}{1 - \gamma(1 - \tau)}. \tag{36}$$

Replacing γ by the value found in (31), this simplifies to

$$u = \frac{\tau}{1 + \tau}. \tag{37}$$

This frictional unemployment derives directly from the rate at which the population is renewed. If workers live forever ($\tau \to 0$), the time spent searching is negligible, but if workers do not expect to live long, this can be substantial. We can however show that if γ as an interior solution when $\lambda \geq \lambda'$, frictional unemployment is never higher than the structural unemployment when $\lambda < \lambda'$.

Proposition 4 *If workers prefer unemployment over a mismatch, and if there exists a value $\lambda' \in (0, 1)$ such that, for all values of $\lambda \geq \lambda'$ workers search for their preferred job in equilibrium, then increasing bargaining power from any $\lambda < \lambda'$ to at least λ' decreases aggregate unemployment.*

Proof. The aggregate level of unemployment is equal to $1 - \alpha$ when $\lambda < \lambda'$ and to $\frac{\tau}{1+\tau}$ when all workers search. The latter is lower whenever $\tau < \frac{1-\alpha}{\alpha}$, which is exactly the condition for an interior solution $\gamma \in (0, 1)$ to exist when $\lambda \geq \lambda'$, in (31). ∎

c. The median voter

Finally, we want to study the value of λ_a, the maximum initial value of λ for the median voter to benefit from an increase in his own bargaining power to λ'.

The median voter is a worker of type a. Hence, in an equilibrium where $\lambda < \lambda'$, the expected surplus of this worker is given by equation (23), $E(U_a | \gamma = 1) = \frac{\lambda V}{\tau}$. His expected equilibrium payoff in an equilibrium where both types of workers search and $\lambda = \lambda'$ is

$$E(U_a) = \gamma \frac{\lambda' V}{\tau} + (1 - \gamma)W^*. \tag{38}$$

Indeed, with probability γ, a worker of type a receives a good match and does not have to search. With probability $1 - \gamma$ he receives the continuation value of searching. Following the same logic as in equation (25) for workers of type b, this continuation value is given by

$$W^* = \frac{r - s}{1 - (1 - \gamma)(1 - \tau)} + \frac{(1 - \tau)\gamma\lambda V}{\tau(1 - (1 - \gamma)(1 - \tau))}. \tag{39}$$

Replacing by the values of γ^* and λ' in (31) and (27), equation (39) can be rewritten as

$$W^* = \frac{r}{\tau} + \frac{s(2\alpha - 1)}{\alpha(1 - \alpha(1 - \tau))}, \tag{40}$$

and therefore, using equation (40), equation (38) rewrite

$$E(U_a|\gamma^*, \lambda') = \frac{r}{\tau} - \frac{s(1 - \alpha)}{\alpha(1 - \tau)} + \frac{s}{1 - \alpha(1 + \tau)}. \tag{41}$$

To understand why the last part of the equation is increasing in s, it is useful to remember that λ' is also increasing in s. Hence, one should not interpret a higher level of search costs as an increase in workers' surplus, but instead as an increase in the necessary bargaining power for search to be an equilibrium.

Using the above result, we can make a proposition similar to the last part of Proposition 2 in the static model.

Proposition 5 *For any pair $\{\hat{\lambda}, \lambda'\}$ such that increasing workers' bargaining power from any $\lambda \in (\hat{\lambda}, \lambda')$ to λ' increases employers' profits, there exists a value $\lambda_a < \lambda'$ such that, for any $\lambda \in (\lambda_a, \lambda')$, workers of type a are better off with bargaining power λ than λ'.*

Proof. To determine the value of λ_a, we need to find the maximum value of λ such that $E(U_a|\gamma = 1, \lambda) \le E(U_a|\gamma^*, \lambda')$. The solution is

$$\lambda_a = \frac{\tau}{V} E(U_a|\gamma^*, \lambda'). \tag{42}$$

By comparing the value of λ_a to the value of λ' found in equation (27), we find that $\lambda' > \lambda_a$ for all values of the parameters, as

$$\lambda' - \lambda_a = \frac{s\tau(1 - \alpha)}{\alpha V(1 - \tau)} > 0. \tag{43}$$

∎

5 Endogenous bargaining

We now come back to the static version of the game and propose to dissociate the bargaining power in two parts. The first part, formal bargaining, corresponds to the λ described above. These general regulations are assumed to be independent of the market structure. The second part, strategic bargaining, depends on the competition among firms. Indeed, the outside option of a worker is not always to not take a job (r), but also to look for another job.

Hence, for a given set of the exogenous parameters v, V, r and s, the bargained wage of a worker of type $i \in (a, b)$ in sector $J \in (A, B)$ is of the form:

$$w_{iJ} = w_i(J, \lambda, \gamma). \tag{44}$$

From the previous sections, we have already assumed that a good match is better than a mismatch ($w_{aA} > w_{aB}$), and that a higher value of λ increases, all other things being equal, the wage of a worker ($\frac{\partial w_i}{\partial \lambda} > 0$). What we add here is the influence of γ: more employers in a workers' preferred sector increases his wage ($\frac{\partial w_a}{\partial \gamma} > 0$ and $\frac{\partial w_b}{\partial \gamma} < 0$). We assume the function to be symmetric, $w_a(A, \lambda, \gamma) = w_b(B, \lambda, 1 - \gamma)$ and $w_a(B, \lambda, \gamma) = w_b(A, \lambda, 1 - \gamma)$. Moreover, we want the absolute impact of the endogenous bargaining power not to decrease when the quality of matching increases ($w_a(A, \lambda, \gamma') - w_a(A, \lambda, \gamma) > w_a(B, \lambda, \gamma') - w_a(B, \lambda, \gamma)$ when $\gamma' > \gamma$) and the bargained wages to leave a positive surplus to firms and workers ($w_a(A, \lambda, \gamma) \leq V$ and $0 \leq w_a(B, \lambda, \gamma) \leq v$).

What we are interested in here is (i) the conditions for both types of workers to search to be an equilibrium, (ii) the impact on firms' profit, and (iii) the preferences of the median voter.

As in the first section, we assume a worker prefers a mismatch over unemployment. We therefore set the payoff of not working to $r = 0$. The condition for a worker of type b to search in the second stage of the game is

$$w_b(B, \lambda, \gamma) - \frac{s}{1 - \gamma} \geq w_b(A, \lambda, \gamma). \tag{45}$$

Assuming $\gamma > \frac{1}{2}$ (we show below it is always the case), if a worker of type b searches when mismatched, a worker of type a also searches (as the functions are symmetric).

In the first period, assuming both types of workers search, the respective expected profits of

firms in sector A and B are given by:

$$E(\pi|A) = \frac{\alpha}{\gamma}(V - w_a(A, \lambda, \gamma))$$

$$E(\pi|B) = \frac{1-\alpha}{1-\gamma}(V - w_b(B, \lambda, \gamma)) \qquad (46)$$

Proposition 6 *With endogenous bargaining power, if both types of workers search in equilibrium: (i) the share of firms in sector A is lower than the share of workers of type a and (ii) the wage of workers of type a is higher than the wage of workers of type b.*

Proof. Start by assuming that $\gamma = \alpha$. In that case, $w_a(A, \lambda, \alpha) > w_b(B, \lambda, \alpha)$. Hence, $E(\pi|A) < E(\pi|B)$ $\forall \gamma \geq \alpha$. Thus, more firms want to enter in sector B. We also know that, if $\gamma = \frac{1}{2}$, $w_a(A, \lambda, \frac{1}{2}) = w_b(B, \lambda, \frac{1}{2})$. Hence, $E(\pi|A) > E(\pi|B)$ $\forall \gamma \in (0, \frac{1}{2})$. It follows that $\gamma^* \in (\frac{1}{2}, \alpha)$, and thus $w_a(A, \lambda, \gamma^*) > w_b(B, \lambda, \gamma^*)$. ∎

This result is reminiscent of Albrecht and Axell (1984): when firms offer different wages and workers have identical productivity, the equilibrium number of employee-per-firm must differ for some firms to have an incentive to offer the higher wage. Now, compare this equilibrium with the one where jobs are provided in sector A only.

The expected profit of a firm is given by:

$$E(\pi|A) = \alpha(V - w_a(A, \lambda, 1)) + (1 - \alpha)(v - w_b(A, \lambda, 1)) \qquad (47)$$

We see that, in the presence of a more diverse market structure, the firms are better off for two reasons. First, because they get a higher matching quality. Second, because the endogenous bargaining power of those workers with a good match decreases, and therefore the wages also decrease. This implies that, as compared to the exogenous case described in the previous sections, the incentives for firms to accept an increase in the formal bargaining power of workers are even higher.

This result has an ambiguous effect on the necessary level of λ for workers of type b to search. Indeed, as compared to the case with only exogenous bargaining power, two things are different. The first difference is that the share of firms in sector b is higher. Hence, the necessary wage for workers of type b to search is lower. The second difference is that the wages of minority workers are lower, hence workers of type b have lower incentives to search. Moreover, when the share of firms in sector b increases, the wage bargained from a mismatch also increases, as $w_b(A, \lambda, 1) < w_b(A, \lambda, \gamma^*)$. Increasing the formal bargaining power still unambiguously increases

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the willingness of minority workers to search, as we have assumed the impact of the endogenous bargaining power not to decrease when the quality of matching increases.

When considering workers of type a, and therefore the median voter, a first result is that the impact of an increased formal bargaining power on the actual wages is ambiguous, as $w_a(A, \lambda, 1) > w_a(A, \lambda, \gamma^*)$. Hence, for a given value of λ, the median voter is better off with firms in sector A only than in the case where both types of workers search. The first reason is that, as above, he does not have to bear search costs. The second is that, because all firms offer his preferred job, his endogenous bargaining power is higher. This is why endogenous bargaining makes the median voter even less likely to accept an increase in formal bargaining power.

A last important point is that, if the share of minority workers is close to $\frac{1}{2}$, and if the endogenous part of wage bargaining is important enough, there may simply be no equilibrium where jobs are offered in sector A only.

Indeed, when all firms offer jobs in sector A, it is a best response for a firm to choose sector B iff:

$$(1 - \alpha)(V - w_b(B, \lambda, 1)) > \alpha(V - w_a(A, \lambda, 1)) + (1 - \alpha)(v - w_b(A, \lambda, 1)). \qquad (48)$$

In this last equation, the expected profit of the firm in sector B consists of a smaller part of the population, with a good match and a lower bargaining power. On the other hand, firms operating in sector A get a combination of workers with a good match but high bargaining power, and of mismatched workers. If this inequality is fulfilled, there are always firms operating in both sectors.

6 Extensions

This section considers several extensions to the baseline model.

6.1 Minimum wage and the outside option

It is interesting to compare bargaining power with two other parameters than can be interpreted as the result of policy decisions: the minimum wage and the outside option r. The tradeoff we are interested in is between search costs (that generate frictional unemployment in the dynamic model) and the quality of matching, and how it affects the expected surplus of the different players. In our model, for a minimum wage w_{min} to have an impact on the economy it must meet the following

condition:

$$w_{min} > max\{\lambda v, r\} \tag{49}$$

We also assume $w_{min} < \lambda V$, so that it applies to mismatches only. The minimum wage has an ambiguous effect on the market incentives to increase the quality of matching (and therefore, in our specification, frictional unemployment). On the one hand, employers have higher incentives to offer jobs in sector B, because the expected surplus they can extract from mismatched employees decreases. If both categories of jobs are offered, and if workers of both types search for a good match, the surplus of a mismatch is not relevant anymore for the expected profit. On the other hand, those employees have lower incentives to look for the right job, because, by condition (49):

$$\lambda V - w_{min} < \lambda V - max\{\lambda v, r\}. \tag{50}$$

This last condition explains the following proposition.

Proposition 7 *An increase in the minimum wage (weakly) decreases the average quality of matching and the average search costs.*

Proof. If both types of workers are looking for the right job, the isoprofit conditions (7) and (8) are unaffected and, therefore, in equilibrium, $\gamma = \alpha$. However, the conditions on workers' bargaining power to search for the right job become more restrictive. In particular, equations (10) and (22) rewrite:

$$\lambda \geq \frac{s + (1 - \alpha)w_{min}}{(1 - \alpha)v} > max\{\lambda', \lambda''\}. \tag{51}$$

Hence, by condition (49), if w_{min} has an impact, it is to modify the conditions leading to equilibrium (c), and therefore to decrease both the share of workers who search and the average quality of matching. ∎

If the minimum wage decreases the quality of matching, the effect on majority type workers is ambiguous, following the logic exposed in the baseline model. However, an increase in the minimum wage that does not decrease the average quality of matching makes workers of type b unambiguously better off. An important assumption is however that $w_{min} \leq v$, so that employers are still willing to pay the minimum wage to mismatched workers.

Proposition 8 *A decrease in the outside option (weakly) increases total employment and has an ambiguous impact on the average quality of matching. If the quality of matching increases, the expected surplus of workers of the majority type unambiguously decreases and the impact on minority workers is ambiguous. If the quality of matching decreases, only workers of the minority type are worse off.*

Proof. Assume the outside option decreases from r to r'. If $r > r' > \lambda v$, the outside option decreases the value of λ'' in condition (22). In this case, decreasing the level of the outside option increases the incentive of minority type workers to look for the right job, but also decreases the expected utility of those workers if $s < r - r'$. Indeed, their expected surplus switches from

$$U_b(L) = r \tag{52}$$

to

$$
\begin{aligned}
E(U_b) &= (1-\gamma)\lambda''V + \gamma U_b(S) \\
&= \lambda''V - \frac{\alpha s}{1-\alpha} \\
&= \frac{(1-\alpha)r' + s}{1-\alpha} - \frac{\alpha s}{1-\alpha} \\
&= r' + s.
\end{aligned} \tag{53}
$$

The expected surplus of workers of the majority type decreases, as their expected search costs increase without compensation. If $r > \lambda v > r'$, the binding condition switches from (22) to (10), with either no unemployment and a decrease in the average quality of matching (if condition (10) is not met) or no unemployment and an increase in the payment of search costs, making both types of workers worse off. Finally, if $\lambda v > r > r'$, the policy has no impact. ∎

This policy is particularly relevant when considering the Spanish example again. A policy of decreased outside option may increase the number of jobs while hurting only minority type workers. This is a popular option, as it corresponds to the taste of the median voter, but it does not solve the main problem: the fact that workers of type b do not find jobs that correspond to their skills.

6.2 Training

Another popular policy is to provide unemployed workers with training in order to make them more productive in existing jobs. We study this policy in the baseline model, assuming workers prefer a mismatch over unemployment, $\lambda v \geq r$. Assume that there is a cost of $c > 0$ (paid by the employer) that allows a worker of type b to be trained, and get the same skills as a worker of type a. Consider a level of bargaining power sufficiently low, $\lambda < \lambda'$, so that $\gamma = 1$ in equilibrium. The payoff of a worker of type a is thus equal to λV, and the one of a worker of type b is λV if he receives the training, and λv if he doesn't. The expected benefit of this training on the profit of firms is equal to $(1 - \lambda)(1 - \alpha)(V - v)$. The cost of providing this training is equal to $(1 - \alpha)c$. Hence, a firm chooses to provide training if

$$c < (1 - \lambda)(V - v), \tag{54}$$

while providing training would be socially optimal if

$$c < V - v, \tag{55}$$

hence a classic holdup problem, where firms do not want to invest enough in training because they receive only a share of its benefits.

In our specification, we know that a better matching quality could be obtained by increasing bargaining power to at least λ', at the social cost of increasing aggregate search costs to s.[12] Hence, training is socially more beneficial (in terms of aggregate surplus) than increasing bargaining power iff

$$1 - \alpha < \frac{s}{c}$$
$$\Leftrightarrow c < \frac{s}{1 - \alpha} \tag{56}$$

If condition (56) is fulfilled, a government maximizing aggregate welfare should subsidize training when either search costs are very high or the minority is very small, $s > (1 - \alpha)(1 - \lambda)(V - v)$ and, using (54), the per-worker subsidy should be equal to $\sigma = \frac{s}{1-\alpha} - (1 - \lambda)(V - v)$ so that firms want to provide training whenever it is welfare improving.

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[12]The aggregate search cost when both types of workers search is equal to $\alpha \frac{1-\gamma}{\gamma} s + (1 - \alpha) \frac{\gamma}{1-\gamma} s$, equal to s as in equilibrium $\gamma = \alpha$.

Finally, we want to estimate when a firm prefers to provide training instead of increasing bargaining power to λ'. This is the case when

$$(1-\alpha)s < (\lambda - \lambda')(V-v). \tag{57}$$

We can thus find λ_t such that $\forall \lambda \leq \lambda_t$, firms prefer to pay training over an increase in λ to λ', by replacing λ' by its equilibrium value in (57),

$$\lambda_t = \frac{\alpha^2 c - 2\alpha c + c + s}{(1-\alpha)(V-v)}. \tag{58}$$

And $\lambda_t > \hat{\lambda}$ iff

$$c < \frac{(V-v)((1-\alpha)(V-v)-s)}{(1-\alpha)((1-\alpha)v+\alpha V)}. \tag{59}$$

which means that when condition (59) is satisfied, for some values of $\lambda \geq \hat{\lambda}$ where increasing workers' bargaining power to λ' would make employers better off, those firms still prefer to subsidize training. We can show that, as the condition in (56) is less restrictive than the condition in (59),[13] whenever such a phenomenon arises, it is socially beneficial.

6.3 More than two sectors

While our setup is a two-types, two-sectors economy, the results could easily be extended to a larger number. Denote by α_i the share of workers of type i, employers will supply such jobs (and those workers will search in equilibrium) when, depending on whether $\lambda v \geq r$, either of those conditions is fulfilled:

$$\alpha_i \geq \frac{s}{\lambda(V-v)} \tag{60}$$

$$\alpha_i \geq \frac{s}{\lambda V - r}. \tag{61}$$

The difference is that the number of tradeoffs increases. Increasing the expected quality of matchings is more costly in terms of search costs when the share of workers of a given type is small. Therefore, it can be socially optimal (in the sense of Propositions 1 and 2) to increase workers' bargaining power up to a certain level (to allow the most represented types of workers to

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[13]Denoting $c' = \frac{s}{1-\alpha}$ and $c'' = \frac{(V-v)((1-\alpha)(V-v)-s)}{(1-\alpha)((1-\alpha)v+\alpha V)}$, we find that $c' \geq c''$ whenever $\alpha < 1 + \frac{sv}{(V-v)(V-v-s)}$, which is always true, as a necessary condition for workers to search at least once is $s < V - v$, and as $\alpha \leq 1$.

search), but without necessarily having all types of workers searching in equilibrium.

6.4 Asymmetric bargaining power

We consider two types of workers with two different levels of bargaining power, λ_a and λ_b. We show that if λ_i increases for one type of workers i only, the marginal benefit these workers get from searching is counterbalanced by the decrease in the equilibrium share of firms in the corresponding sector I if both types of workers search. We then show that this negative impact is less important if the bargaining power of workers of type i is lower than the bargaining power of workers of type j.

Assume first the two types of workers search in equilibrium. The expected profit of firms given in (7) and (8) becomes

$$E(\pi|A) = \frac{\alpha}{\gamma}(1 - \lambda_a)V \tag{62}$$

$$E(\pi|B) = \frac{1 - \alpha}{1 - \gamma}(1 - \lambda_b)V, \tag{63}$$

and both are equal iff

$$\gamma^* = \frac{\alpha(1 - \lambda_a)}{(1 - \lambda_a) + (1 - \alpha)(\lambda_a - \lambda_b)}, \tag{64}$$

so that $\gamma^* > \alpha$ whenever $\lambda_b > \lambda_a$.

The expected benefit from searching for a mismatched worker of type b is given by

$$B_b(\lambda_a, \lambda_b) = \lambda_b(V - v) - \frac{s}{1 - \gamma}, \tag{65}$$

so that if bargaining powers are identical, $\lambda_a = \lambda_b = \lambda$, this marginal benefit is given by $\frac{\partial B(\lambda)}{\partial \lambda} = V - v$. Using (64) in (65) yields

$$B_b(\lambda_a, \lambda_b) = \lambda_b(V - v) - s(1 + \frac{\alpha(1 - \lambda_a)}{(1 - \alpha)(1 - \lambda_b)}). \tag{66}$$

Hence, the increase in the benefit from searching when increasing λ_b alone is

$$\frac{\partial B_b(\lambda_a, \lambda_b)}{\partial \lambda_b} = (V - v) - \frac{\alpha s(1 - \lambda_a)}{(1 - \alpha)(1 - \lambda_b)^2}. \tag{67}$$

The first term is the benefit from search when the increase is symmetric, and the second term is the negative effect due to the decrease in the equilibrium share of firms in sector b. This negative term is smaller when the bargaining power of workers of type a is higher, and bigger when λ_b is already high. In the particular example of dual labor markets, this result implies that increasing the bargaining power for the workers with the lowest λ limits the negative impact in terms of equilibrium share of firms in their sector. We can derive similar results for workers of type a.

7   Conclusion

We have shown that, when considering the labor market as a game of horizontal matching between workers and employers, insufficient bargaining power for workers can lead to a reverse holdup problem: employers are not offering the efficient job diversity, as they know workers will not search for the right job. When the economy is doing well, the wages of mismatched workers are low, but there is neither frictional nor structural unemployment. However, when the economy is doing badly, mismatched workers are not productive enough anymore. In that case, increasing their bargaining power may lead to more jobs, better jobs, and higher profits for employers.

Although increasing workers' bargaining power emerges as a credible strategy to improve horizontal matching, there is a risk associated with it. Even when it creates better jobs and leads to higher expected wages, a majority of workers may lose expected utility because their search costs increase in this more dynamic economy. This may lead a majority of workers to oppose a policy that would increase their own bargaining power. These workers would enjoy a higher welfare in a world with fewer opportunities for newer workers or workers with new skills. Interpreting the game as 'older' and 'new' workers and sectors, it is worth noting that waiting for the minority skill to be a majority does not help. Indeed, if they become a majority, workers with 'new' skills will oppose any increase in the bargaining power that would help create jobs in an even 'newer' sector.

The objective of industrial policy is to switch production to the sectors that make the most of the skills of all types of worker as a way of increasing the productivity of the economy. The objective of labor policy is to ensure that all workers types eventually have employment opportunities. Our model allows us to explain why a standard democratic process may fail to reconcile both objectives, as currently observed in many European countries unable to modernize and reduce unemployment. We show that the fear of a majority of workers to see their search costs increase can lead them to impose an inefficient industrial policy to achieve the labor policy goals, because there is no way for the minority type to move alone unless all workers accept an increase in their

bargaining power. The key to reconciling the incentives of the median voter with the optimal in-dustrial policy may come from social policy. In particular, a combination of increased bargaining power for all workers (i.e. their ability to explicitly link their salaries to profits in upswings as well as in downswings) with a lump-sum transfer from minority workers to majority workers may help. This is because it compensates the majority for their higher search costs in a more efficient economic structure while increasing employment opportunities for minority workers. In this quite common setting, industrial, labor and social policy can, thus, not be designed independently, if reforms are to be supported by the median voter. And this may be what many European countries have underestimated.

Our theoretical case for increased bargaining power for specific workers groups is echoed in many current policy debates, in particular debates surrounding the ways to offset the shrinking of the middle class or the increasing gap between the returns to capital and labor and the resulting erosion of the labor share in the national income in many countries. The renewal of the case for profit sharing, for more cooperative production structures and for a restructuring of trade unions' mandate to close the gaps between insiders and outsiders are simply different ways of targeting the improvements of bargaining power to various stakeholders. Although their relative effectiveness is clearly an empirical matter and will depend on the institutional context in which the instruments are implemented, this paper suggests that they are all steps in the right direction if countries are to make the most of the potential offered by both their labor and product markets.

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Appendix

1 Proof of Proposition 1

This proposition relates to a world without structural unemployment. From Table 1, we know we have to consider two equilibria: when bargaining power is low, all employers offer jobs in sector *A*, and when bargaining power satisfies condition (10), employers offer jobs in both sectors. The two equilibria can be simultaneously SPNE. This is the case for every $\frac{s}{(1-\alpha)(V-v)} \leq \lambda$, but in this case only the equilibrium with diversity is on the Pareto frontier for firms, as the expected profit is higher. To prove point (ii) of the proposition, we need to determine $\hat{\lambda}$ such that firms are indifferent between $\hat{\lambda}$ and λ'. This is the case when their expected profit is equal:

$$(1 - \lambda')V = \alpha(1 - \hat{\lambda})V + (1 - \alpha)(1 - \hat{\lambda})v. \tag{68}$$

As $V > v$, we can already see from equation (68) that, for every value of λ', there exist values of λ such that the profit increases when workers' bargaining power increases from $\lambda < \lambda'$ to λ'. However, it is not the case when λ is too low. Replacing λ' by its expression, the exact condition is given by:

$$\hat{\lambda} = 1 - \frac{((1-\alpha)(V-v) - s)V}{(\alpha V + (1-\alpha)v)(1-\alpha)(V-v)}. \tag{69}$$

Thus, (i) For any $\lambda' \in (0,1)$, $\exists\, \hat{\lambda} < \lambda'$ such that increasing workers' bargaining power from any $\lambda \in (\hat{\lambda}, \lambda')$ to λ' increases the expected profit of employers, (ii) λ' increases with α, (iii) $\hat{\lambda}$ increases with λ' and α as from equation (68) and (iv) when α increases, the gap between $\hat{\lambda}$ and λ' decreases (as $\frac{d\hat{\lambda}}{d\lambda'} = \frac{V}{\alpha V + (1-\alpha)v} > 1$).

The last part (iii) of the proposition is determined by the value of $\lambda = \lambda_a$ such that workers of the majority type a are indifferent between λ_a and λ'. This value is given by solving the following equality:

$$\lambda_a V = \lambda' V - \frac{s(1-\gamma)}{\gamma}. \tag{70}$$

Replacing λ' by its value and γ by its equilibrium value α when $\lambda \geq \lambda'$ yields

$$\lambda_a V = \frac{sV}{(1-\alpha)(V-v)} - \frac{(1-\alpha)s}{\alpha}$$
$$\Leftrightarrow \lambda_a = \frac{s\left(\alpha^2 v - 2\alpha v + v - \alpha^2 V + 3\alpha V - V\right)}{(1-\alpha)\alpha V(V-v)}$$
$$\Leftrightarrow \lambda_a = \frac{s(\alpha V - (1-\alpha)^2(V-v))}{\alpha V(1-\alpha)(V-v)} \tag{71}$$

with $\lambda_a < \lambda'$. Intuitively, the higher the search cost s, the higher the risk that those majority workers lose from the economic diversification. As the median voter is, by definition, of the majority type, he will oppose any increase of bargaining power that decreases his own welfare.

2 Threshold values of λ when $\lambda v < r$

Second stage In the second stage, a mismatched worker of type A searches iff

$$r \leq \lambda V - \frac{s}{\gamma}$$
$$\lambda \geq \frac{\gamma r + s}{\gamma V}. \tag{72}$$

Similarly, a mismatched worker of type b searches iff

$$\lambda \geq \frac{(1-\gamma)r + s}{(1-\gamma)V} \tag{73}$$

First stage In the first stage, if no worker searches, $\gamma = 1$, as the expected profit of an employer is higher in sector A ($\alpha > \frac{1}{2}$). This equilibrium exists for all parameter values, and yields expected profit

$$E(\pi|A) = \alpha(1-\lambda)V. \tag{74}$$

If both types of workers search, the expected profit of employers is similar to the case where $\lambda v > r$, and the equilibrium share of employers of sector A is $\gamma = \alpha$. Hence, the threshold value for search to be an equilibrium is

$$\lambda \geq \frac{(1-\alpha)r + s}{(1-\alpha)V} = \lambda''. \tag{75}$$

If only workers of type b search, the expected share of employers in sector A is given by equating the expected profits,

$$E(\pi|A) = \alpha(1-\lambda)V$$
$$E(\pi|B) = \frac{1-\alpha}{1-\gamma}V. \qquad (76)$$

Hence, in equilibrium $1 - \gamma = \frac{1-\alpha}{\alpha}$ and $\gamma = \frac{2\alpha-1}{\alpha}$. The first condition for this to be an equilibrium is that workers of type b search, this is:

$$\lambda \geq \frac{\frac{1-\alpha}{\alpha}r+s}{\frac{1-\alpha}{\alpha}V}$$
$$\Leftrightarrow \lambda \geq \frac{(1-\alpha)r + \alpha s}{(1-\alpha)V}. \qquad (77)$$

The second condition is that workers of type a do not search, this is:

$$\lambda < \frac{\frac{2\alpha-1}{\alpha}r+s}{\frac{2\alpha-1}{\alpha}V} \qquad (78)$$
$$\Leftrightarrow \lambda < \frac{\alpha s + (2\alpha-1)r}{(2\alpha-1)V} \qquad (79)$$

In this case, all unemployment comes from workers of type a who are initially mismatched, hence unemployment is equal to

$$u = \alpha(1-\gamma) = (1-\alpha). \qquad (80)$$

3 Proof of Proposition 2

If workers choose unemployment over a mismatch, this implies that $\lambda v < r$. Therefore, increasing the workers' bargaining power to

$$\frac{s + (1-\alpha)r}{(1-\alpha)V} = \lambda'' \qquad (81)$$

always increases the total number of jobs. For it to increase the expected profit, an additional condition (the intuition is similar to equation (68)) applies:

$$(1 - \lambda'')V \geq \alpha(1-\lambda)V \qquad (82)$$

and, replacing λ'' by its expression yields the following condition on $\hat{\lambda}''$:

$$\hat{\lambda}'' = 1 - \frac{(1-\alpha)(V-r) - s}{\alpha(1-\alpha)V}, \qquad (83)$$

such that for any $\lambda'' \in (0,1)$, $\exists\, \hat{\lambda}'' < \lambda''$ such that increasing workers' bargaining power from any $\lambda \in (\hat{\lambda}'', \lambda'')$ to λ'' increases the expected profit of employers. As in the world without structural unemployment, both $\hat{\lambda}''$ and λ'' increase with α, and $\frac{d\hat{\lambda}''}{d\lambda''} > 1$.

The last part (iii) of the proposition is determined by the value of $\lambda = \lambda_a''$ such that workers of the majority type a are indifferent between λ_a and λ''. This value is given by solving the following equality:

$$\lambda_a'' V = \lambda'' V - \frac{s(1-\gamma)}{\gamma}. \tag{84}$$

Replacing λ'' by its value and γ by its equilibrium value α when $\lambda \geq \lambda''$ yields

$$\lambda_a'' V = \frac{(1-\alpha)r+s}{1-\alpha} - \frac{(1-\alpha)s}{\alpha}$$

$$\Leftrightarrow \lambda_a'' = \frac{\alpha r - \alpha^2 r - \alpha^2 s + 3\alpha s - s}{(1-\alpha)\alpha V}$$

$$\Leftrightarrow \lambda_a'' = \frac{(1-\alpha)\alpha r + ((3-\alpha)\alpha - 1)s}{(1-\alpha)\alpha V} \tag{85}$$

4 Dynamic setting with on-the-job search

This extension provides the solution to the dynamic setting, in the case where workers prefer a mismatch over unemployment, $\lambda v \geq r$. The search is on-the-job so that a worker that decides to search while being mismatched still receives the wage of the mismatch, but bears a per-period search cost s.

Workers

If no one searches and $\gamma = 1$ Every worker accepts an offer in the first period. Hence, a worker of type a receives an expected lifetime surplus of

$$E(U_a) = \frac{\lambda V}{\tau}, \tag{86}$$

and a worker of type b

$$E(U_a) = \frac{\lambda v}{\tau}. \tag{87}$$

If all workers search and $\gamma \in (0,1)$

Using the same reasoning as in the static setting, we know that if $\gamma > \frac{1}{2}$, when workers of type b search, workers of type a always search.

A worker of type b searches after being matched with a firm in sector A if the expected surplus from searching is higher than the surplus of keeping the "wrong" job. Denote by W^* the continuation value of searching,

$$W^* = \lambda v + (1-\tau)(1-\gamma)\frac{\lambda V}{\tau} - s + \gamma(1-\tau)W^* \tag{88}$$

While searching, a worker keeps his wage λv. This worker survives into the next period with a probability $1-\tau$ and receives a good match after searching once with a probability $1-\gamma$. The continuation value of a good match is $\frac{\lambda V}{\tau}$, and the search cost is s. Finally, the worker survives and is matched again with a firm of the wrong type with probability $(1-\tau)\gamma$.

For a worker to search, the continuation value W^* must be higher than the expected wage while staying in the wrong job $\frac{\lambda v}{\tau}$. Hence, using equation (88) the condition for workers of type b to search is

$$\lambda v(1-\gamma)(1-\tau) \leq (1-\gamma)(1-\tau)\lambda V - s\tau,$$

which gives us the condition λ' for workers of type b to search,

$$\lambda \geq \frac{s\tau}{(1-\gamma)(1-\tau)(V-v)} = \lambda' \tag{89}$$

Employers

When both types of workers search

The expected profit of a firm in sector A is composed of two parts. The first part is the workers of type a who have found the firm after searching (and keep their jobs until they disappear of the market). Those workers provide a per period surplus $(1-\lambda)V$ to the employer. The second part is the workers of type b who keep their (bad) job until they find a good match (assuming they search at every period). Those workers provide a per period surplus $(1-\lambda)v$ to the firm. Hence, the expected surplus received by a firm operating in sector A is

$$E(\pi|A) = \tau[(\frac{\alpha}{\tau} + \frac{\alpha(1-\gamma)(1-\tau)}{\tau} + \frac{\alpha(1-\gamma)^2(1-\tau)^2}{\tau} + ...)(1-\lambda)V +$$
$$((1-\alpha) + (1-\alpha)(1-\tau)\gamma + (1-\alpha)(1-\tau)^2\gamma^2 + ...)(1-\lambda)v]. \tag{90}$$

This becomes

$$E(\pi|A) = \frac{\alpha(1-\lambda)V}{\tau(1-(1-\gamma)(1-\tau))} + \frac{(1-\alpha)(1-\lambda)v}{1-(1-\tau)\gamma}. \tag{91}$$

Following the same logic, we find the expected profit of firms in sector B,

$$E(\pi|B) = \frac{(1-\alpha)(1-\lambda)V}{\tau(1-(1-\tau)\gamma)} + \frac{\alpha(1-\lambda)v}{1-(1-\gamma)(1-\tau)}. \tag{92}$$

The equilibrium share of firms in sector A, γ^* is given by equating the two expressions, $E(\pi|A) = E(\pi|B)$. This yields

$$\gamma^* = \frac{\alpha - \tau(1-\alpha)}{1-\tau}, \tag{93}$$

identical to the value found in the main part of the paper, when $\lambda v < r$.

When no worker searches

As there is a constant stock of workers of mass 1 and workers do not search, the expected profit of a firm is

$$E(\pi|A, \gamma = 1) = \frac{\alpha(1-\lambda)V + (1-\alpha)(1-\lambda)v}{\tau}. \tag{94}$$

Firms benefit from a increase in workers' bargaining power

We want to determine the value of $\lambda = \hat{\lambda}$ such that firms are better off with $\lambda = \lambda'$ than with $\lambda \in (\hat{\lambda}, \lambda')$. To do so, it is useful to first determine the equilibrium profit $E(\pi|A) = E(\pi|B)$, when $\gamma = \gamma^*$ and $\lambda = \lambda'$. Using the values found above, this simplifies to

$$E(\pi|\gamma^*, \lambda') = \frac{V}{\tau} - \frac{(V-v)+s}{1+\tau} - \frac{s(\alpha V + (1-\alpha)v)}{(V-v)((1-\alpha)-\alpha\tau)}, \tag{95}$$

and $\hat{\lambda}$ is found as the solution to $E(\pi|A, \gamma = 1) = E(\pi|\gamma^*, \lambda')$. This simplifies to

$$\hat{\lambda} = 1 + \frac{s\tau}{(V-v)(1-\alpha(\tau+1))} - \frac{V - s\tau + \tau v}{(v+\alpha(V-v))(\tau+1)} \tag{96}$$

If there is an interior solution $\lambda' \in (0,1)$, we can show that $\hat{\lambda} < \lambda'$. Indeed, using equations (89) and (96) $\hat{\lambda} < \lambda'$ as long as

$$s \leq \frac{(V-v)(1-\alpha(1+\tau))}{\tau}, \tag{97}$$

and we can show, using equation (89) that there exists an interior solution $\lambda \in (0,1)$ whenever $s \leq \frac{(V-v)(1-\gamma)(1-\tau)}{\tau}$.

Finally, we want to find the value of λ_a, the maximum initial value of λ for the median voter

to benefit from an increase in his own bargaining power to λ'.

The median voter is a worker of type a. Hence, in an equilibrium where $\lambda < \lambda'$, the expected surplus of this worker is given by equation (86), $E(U_a|\gamma = 1) = \frac{\lambda V}{\tau}$. His expected equilibrium payoff in an equilibrium where both types of workers search and $\lambda = \lambda'$ is

$$E(U_a) = \gamma \frac{\lambda' V}{\tau} + (1 - \gamma)W^*. \tag{98}$$

Indeed, with probability γ, a worker of type a receives a good match and does not have to search. With probability $1 - \gamma$ he receives the continuation value of searching. Following the same logic as in equation (88) for workers of type b, this continuation value is given by

$$W^* = \frac{\lambda v}{1 - (1 - \gamma)(1 - \tau)} + \frac{(1 - \tau)\gamma \lambda V}{\tau(1 - (1 - \gamma)(1 - \tau))} - \frac{s}{1 - (1 - \gamma)(1 - \tau)}. \tag{99}$$

Replacing by the values of γ^* and λ', equation (99) rewrites

$$W^* = \frac{s(\alpha V - (1 - \alpha)(V - v))}{\alpha(V - v)(1 - \alpha(1 + \tau))}, \tag{100}$$

and therefore, using equation (100), equation (98) rewrite

$$E(U_a|\gamma^*, \lambda') = \frac{sV}{(V - v)(1 - \alpha(\tau + 1))} - \frac{(1 - \alpha)s}{\alpha(1 - \tau)}. \tag{101}$$

To determine the value of λ_a, we need to find the maximum value of λ such that $E(U_a|\gamma = 1, \lambda) \leq E(U_a|\gamma^*, \lambda')$. The solution is

$$\lambda_a = \frac{s\tau}{V}\left(\frac{V}{(V - v)(1 - (1 + \tau)\alpha)} - \frac{(1 - \alpha)}{\alpha(1 - \tau)}\right). \tag{102}$$

By comparing the value of λ_a to the value of λ' found in equation (89), we find that $\lambda' > \lambda_a$ for all values of the parameters, as

$$\lambda' - \lambda_a = \frac{s\tau(1 - \alpha)}{\alpha V(1 - \tau)} > 0. \tag{103}$$